

# Binomial and Normal Distribution

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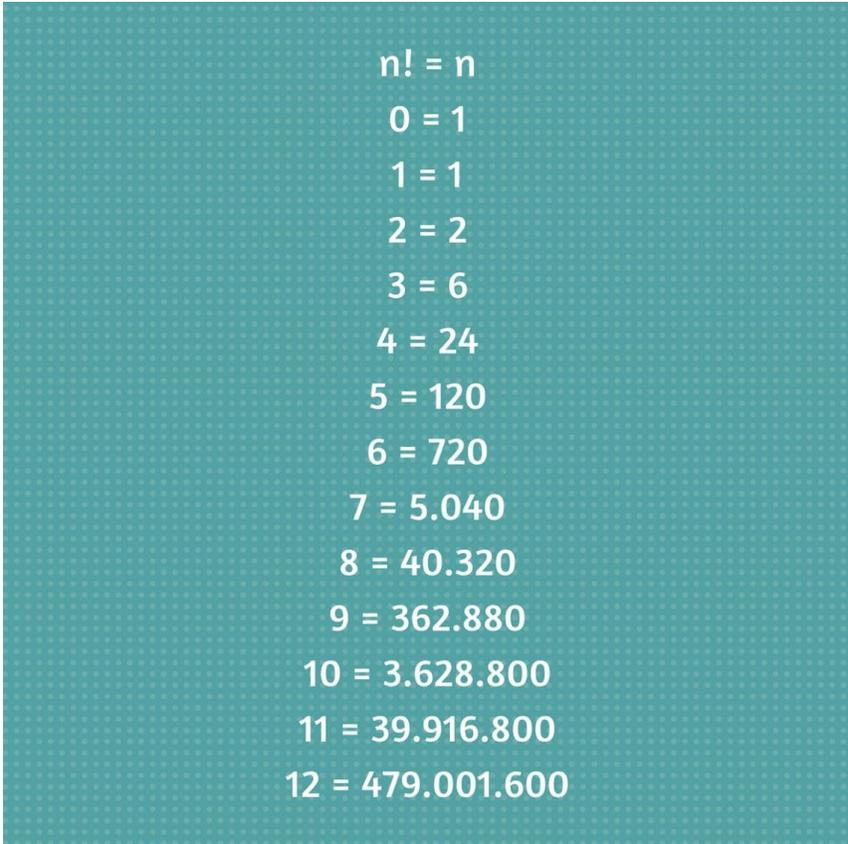
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# Factorials

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- A factorial is a symbol to simplify equations
- $n!$  is equal to the product of the integers from 1 to  $n$
- For example,  $4! = 4 * 3 * 2 * 1 = 24$
- $5! = 5 * 4 * 3 * 2 * 1 = 120$

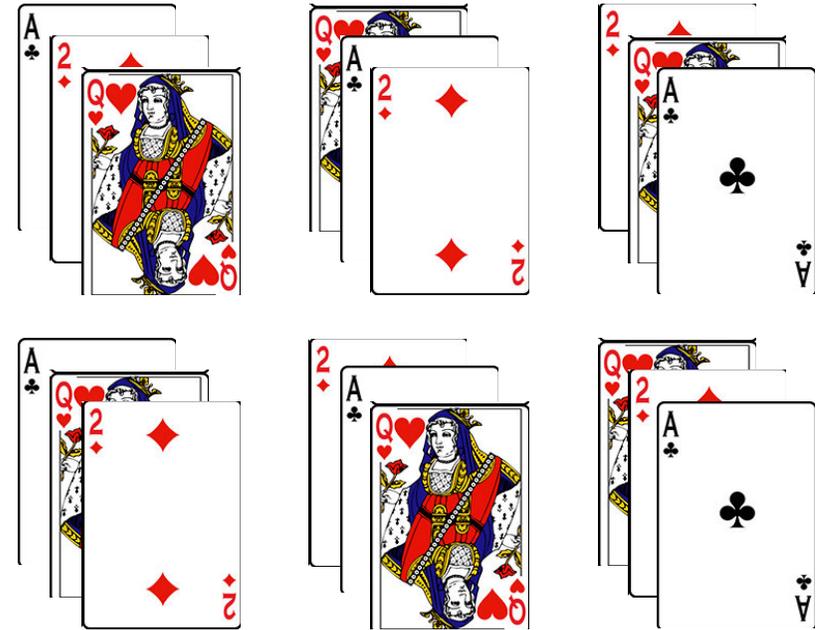


$n!$	$= n$
0	= 1
1	= 1
2	= 2
3	= 6
4	= 24
5	= 120
6	= 720
7	= 5.040
8	= 40.320
9	= 362.880
10	= 3.628.800
11	= 39.916.800
12	= 479.001.600

# Factorials

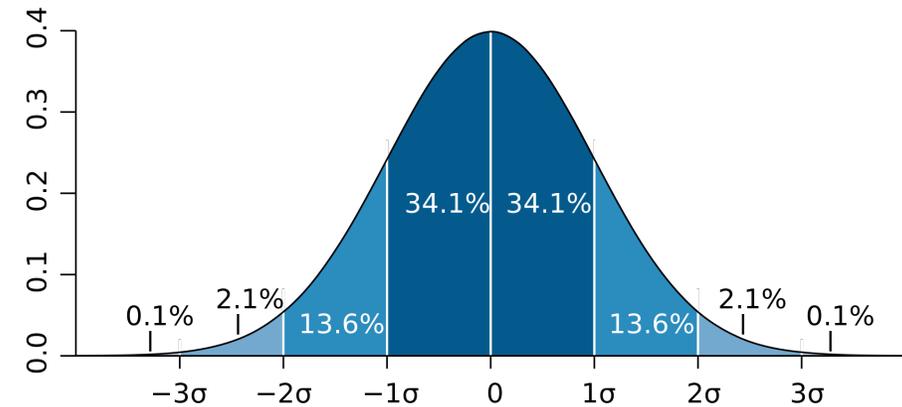
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- The best way to think about factorials is the ways of arranging objects.
- If we have 3 playing cards, then there are 6 different ways we can lay those cards out, this is equal to  $3!$  Which is 6
- This is why  $0!$  is 1 because there is only 1 way to arrange no cards.



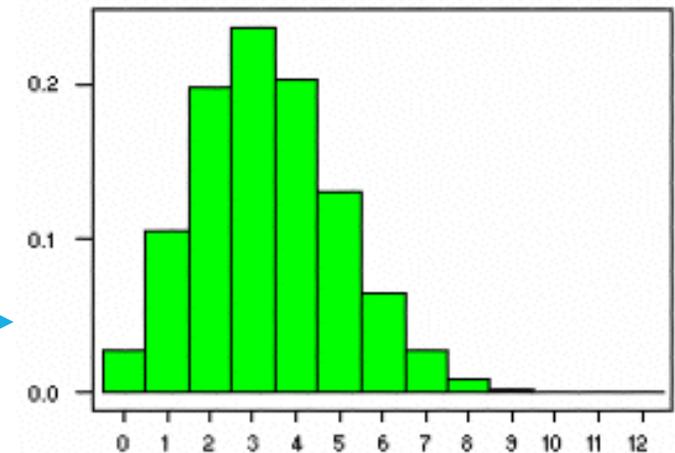
# Binomial & Normal Distribution

- Binomial and normal distributions are both just methods used to model real-world randomness and uncertainty in a mathematically useful way.
- They are helpful in making predictions, understanding variability and decision making.



Normal

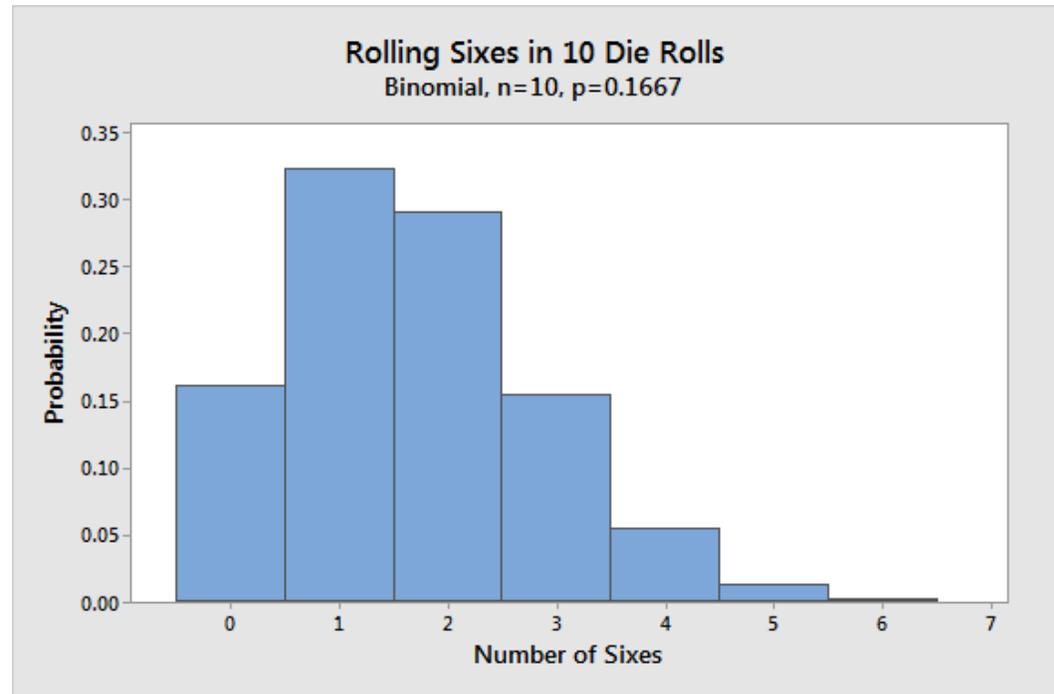
Binomial



# Binomial Distribution

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- A binomial distribution is a **discrete** probability distribution
- This means the values are countable and separate
- Models the probability of something happening in a set amount of trials
- Example: flipping a coin 10 times and counting how many times heads appears



# Binomial Distribution Formula

- Follows the rule:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

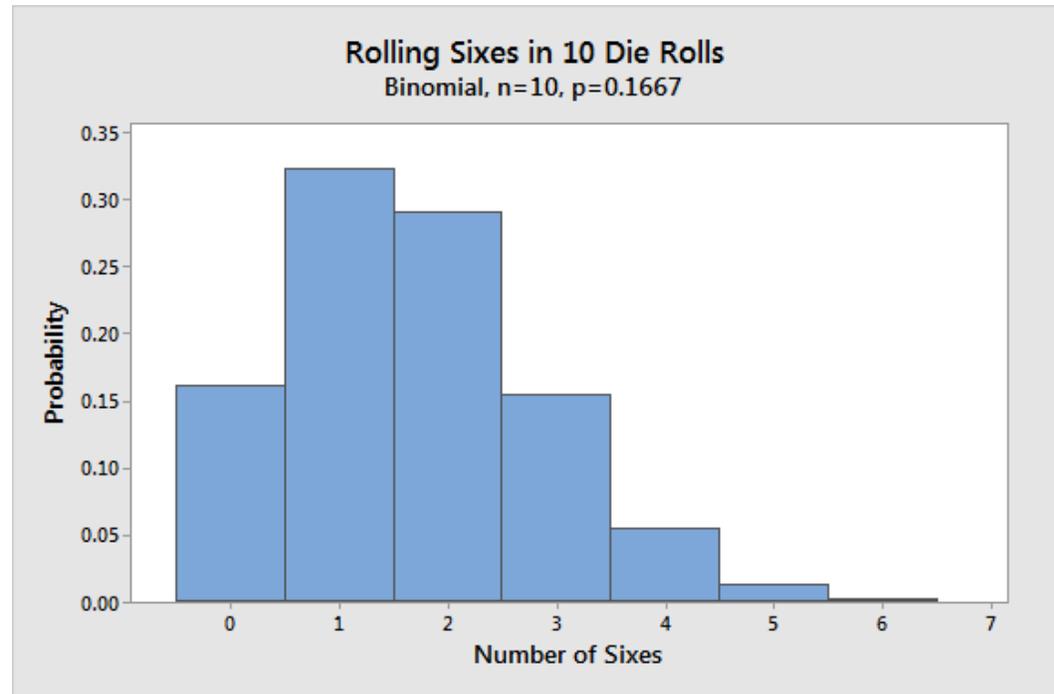
- Where:

$k$  is the amount of successes

$n$  is the number of trials

$p$  is the probability of success in a single trial

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$



# Probability of success (p)

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- We can work out the probability of success in an individual test using the equation:

$$p = \frac{\textit{Number of Success}}{\textit{Total number of trials}}$$

Example: if we flip a coin 10 times and it lands on heads 6 times the probability for heads is  $6/10 = 0.6$

# Example of solving a binomial question

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- A power company manufactures **circuit breakers**, each breaker has a **5% probability** of being faulty. If a technician tests 10 randomly selected breakers, what is the probability that exactly 2 of them are faulty?



# Example of solving a binomial question

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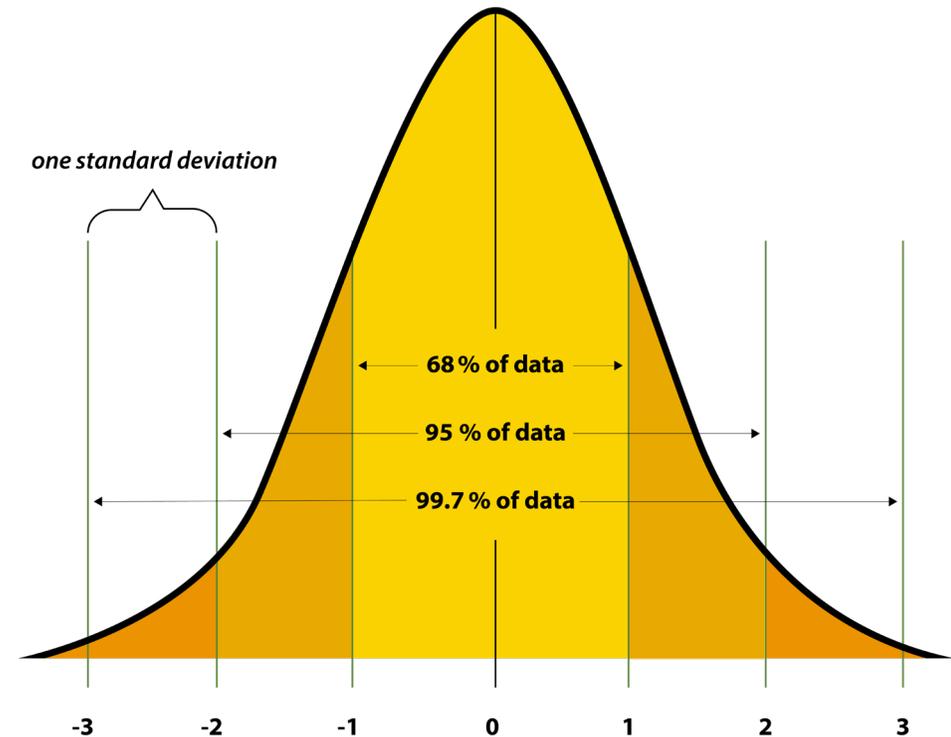
- Identify the given values:
- Total trials:  **$n = 10$**
- Probability of success:  **$p = 0.05$**
- Desired successes:  **$k = 2$**



# Normal Distribution

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- A normal distribution is a continuous (Gaussian) probability distribution
- This means the values can be anything within a range
- Takes the form of a symmetrical bell-shaped curve defined by mean and standard deviation
- Example: exam scores will follow a normal distribution allowing universities to set grading curves





# Reading a z-table

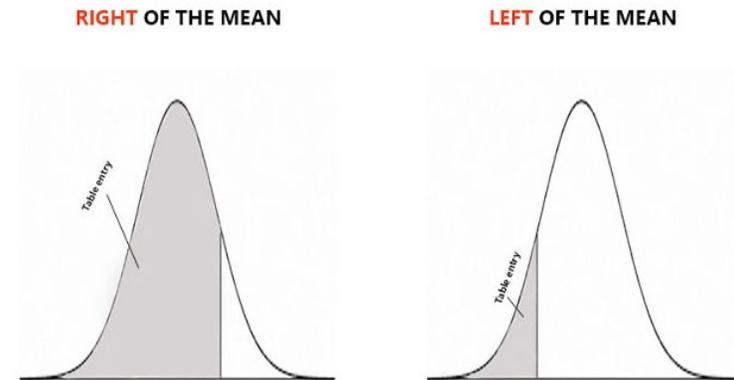
- Z-tables look complicated but they're easy
- All you do is append the column to the row
- example, for example for  $z = -2.51$  we would go to the tenth row (-2.5) then the second column (.01)

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

# Positive vs Negative Z-table

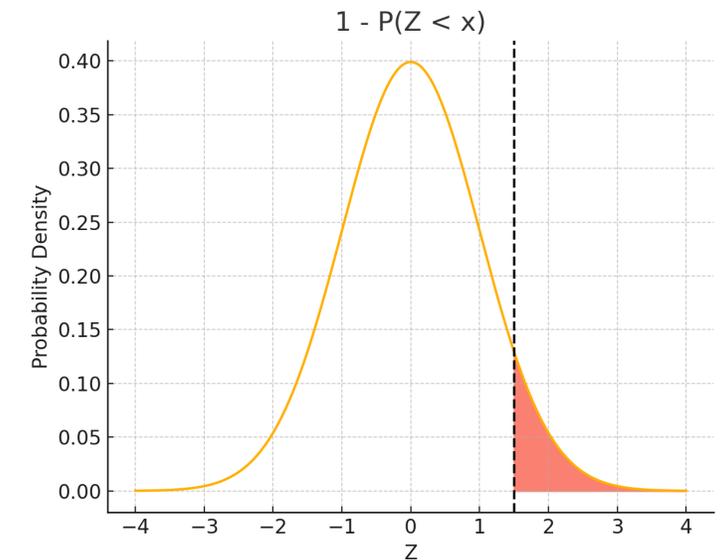
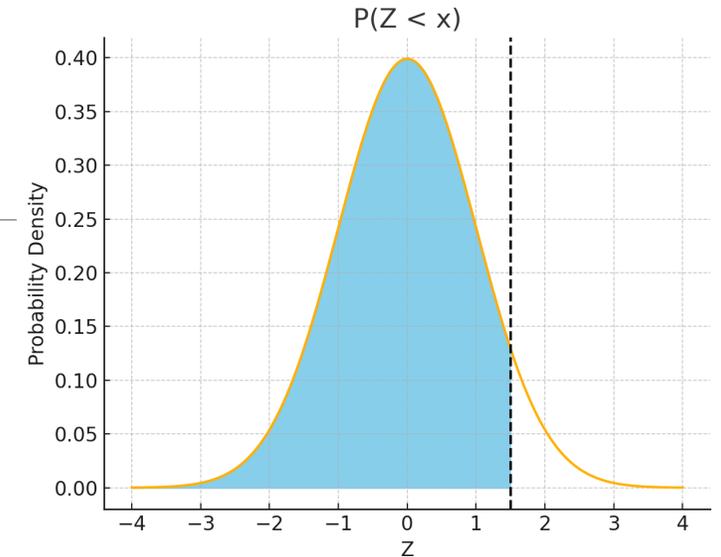
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- Use the positive Z-table when:
  - Your Z-score is greater than 0
  - You are finding probabilities to the left of a positive Z-score
- Use the negative Z-table when:
  - Your Z-score is less than 0
  - You are finding probabilities to the left of a negative Z-score



# Using Symmetry in Z-Score Calculations

- The standard normal distribution is symmetric around  $Z = 0$
- This means:
  - $P(Z < -x) = 1 - P(Z < x)$
- We use symmetry when:
  - A Z-table only gives left-tail values (i.e.,  $P(Z < x)$ )
  - We need the right-tail area,  $P(Z > x)$
- This saves needing a separate negative Z-table



# Probability Density Formula

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- The second equation to do with Normal distribution is the probability density function:

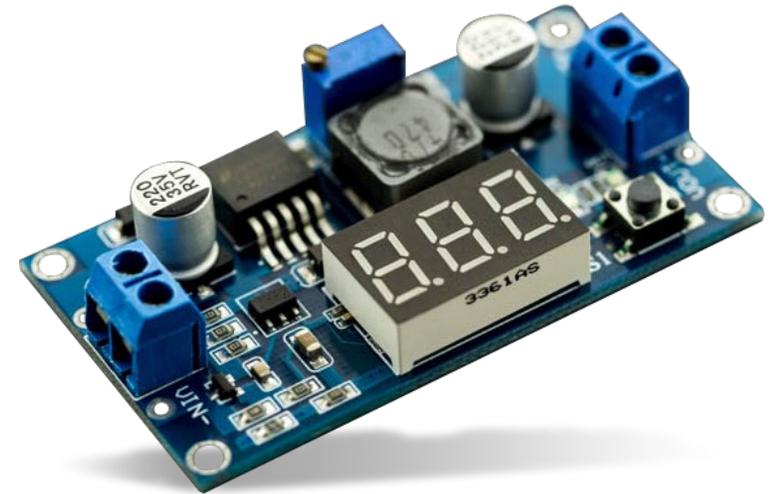
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Where:
  - $x$  is the value we are trying to find (e.g a height of 175cm)
  - $\mu$  is the mean average around which all data is centered (the peak of the bell curve)
  - $\sigma$  is the standard deviation (how far apart the data is)
- Used when determining how dense probability is at a single point, for calculation the relative likelihood of different values
- The density will not give us a probability, it will tell us how likely the value is in that region

# Example of solving a normal distribution (z-score)

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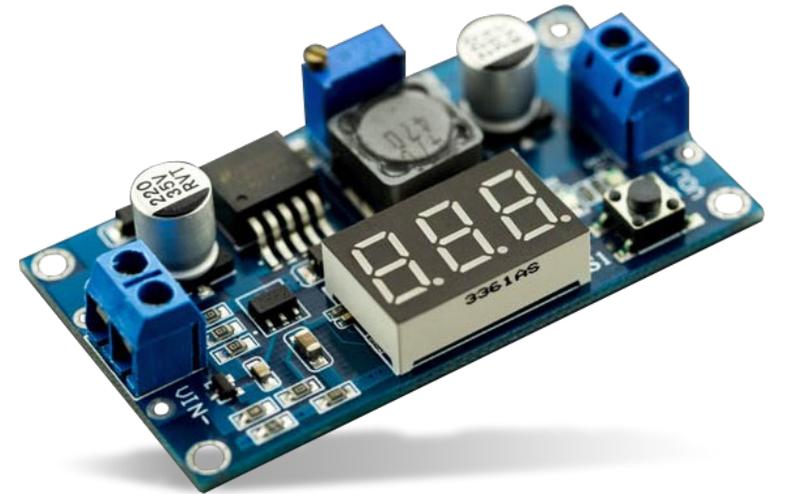
- A component has a mean voltage output of 12v ( $\mu = 12$ )
- They are built with a specification standard deviation of 0.5v ( $\sigma = 0.5$ )
- A component will have to be thrown out if it has an output voltage **below 11v**, what is the probability that a randomly chosen power supply will trigger the alarm?



# Example of solving a normal distribution (z-score)

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- Step 1 is to define the problem:
  - $P(X < 11)$
- Step 2 is to put the values into the z-score equation
- $$Z = \frac{X - \mu}{\sigma} = \frac{11 - 12}{0.5} = -2$$



# Example of solving a normal distribution (z-score)

- Step 3 is to find the probability from the z-table
- $Z = -2.00$
- Therefore probability = 0.0228

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
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-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233

# Example of solving a normal distribution (Probability Density Function)

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- Suppose the heights of students in a class are normally distributed with:
- Mean = 170cm
- Standard Deviation = 5cm
- What is the probability density of a student having a height of 172cm

# Example of solving a normal distribution (Probability Density Function)

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- Step 1: Write out the formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Step 2: Plug in values

$$f(172) = \frac{1}{5\sqrt{2\pi}} e^{-\frac{(172-170)^2}{2*5^2}}$$

# Example of solving a normal distribution (Probability Density Function)

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- Step 3: Simplify the equation

$$f(172) = \frac{1}{5\sqrt{2\pi}} e^{-\frac{(2)^2}{50}} = \frac{1}{5\sqrt{2\pi}} e^{-0.08}$$

- Step 4: Calculate components

$$f(172) = \frac{1}{5 * 2.5066} * 0.9231 = 0.0735$$

- Step 5: Interpret the results

The probability density at 172cm is 0.0735. This means that the relative likelihood of a student being exactly 172cm tall is 0.0735, bear in mind this is not the true probability, just an idea of how likely the height is in that ballpark

# Your Turn

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- **Question 1:** The heights of a population of adults follow a normal distribution with a **mean of 170 cm** and a **standard deviation of 7 cm**.
  - a) Calculate the Z-score for a person with a height of 178 cm.
  - b) Work out the probability of having the height 178cm.
  - b) Using the PDF formula, calculate the probability density for a person with a height of 178 cm.
- **Question 2:** The test scores of a class follow a normal distribution with a **mean score of 75** and a **standard deviation of 8**.
  - a) A student scored 65. Calculate the Z-score.
  - b) Interpret the result: Is this score below average? How far is it from the mean in terms of standard deviations?